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STATISTICS AND PROBABILITY APPLIED TO CONTROL SYSTEM ANALYSIS

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During the period 30 Jun 72 through 31 Aug 73 this research program consisted of the principal investigator, two research associates, and a research aide, resulting in 3 publications and 3 technical reports. The research topics include - Variational Methods in Statistics, Inference in Markov Dependent Firing Distribution, Paired Comparisons, Sampling theory and its applications to the development of new sampling strategy for monitoring air pollutants and Pareto distribution.

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Final Report

STATISTICS AND PROBABILITY APPLIED TO CONTROL SYSTEM ANALYSIS

June 30, 1972 -- August 31, 1973

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INTRODUCTION

With the support of the U.S. Air Force Office of Scientific Research Grant No. AFOSR-72-2329, research was undertaken to study some problems in statistics and probability with special reference to the analysis of control systems. The following areas were studied during the period of June 30, 1972 to August 31, 1973 and the results obtained were communicated to the agency. The list of publications, reports and manuscripts arising from the project are given later. Other research activities of the Principle Investigator and other staff are also mentioned.

The major problem areas studied are described below.

1. Variational Methods in Statistics.

Classical as well as modern variational methods with applications in statistics and stochastic control are surveyed. Some of the early work in the field has been reported in the technical reports by Rustagi (1973).

2. Inference in Markov Dependent Firing Distribution.

The study of the waiting time distribution related to a 2 x 2 Markov chain has been made by the Principle Investigator and his associates in the past few years. During the year of the project, the paper on unbiased estimation and test of independence of the above distribution was prepared and study of various other aspects still continues, see Bai and Rustagi (1973).

3. Paired Comparisons and Miscellaneous Problems.

The work performed on paired comparisons is given in the technical report of Gupta and Singh (1973) and a paper by Singh (1973). The Pareto distribution and associated statistical inference, specially in the sequential case, was studied by Wang (1973).

4. Sampling Theory and its Applications to the Development of a New Sampling Strategy for Monitoring Air-Pollutants.

Let $C_i(t)$, $i = 1,2,\ldots,k$ be the concentration of a pollutant i in the environment such as carbon monoxide, oxidant, particulate matter, sulfur dioxode. Then the vector $C(t) = [C_1(t), C_2(t),\ldots,C_k(t)]$ represents the pollution level at time t. In general C(t) is assumed to be a stochastic process and various attempts have been made to study this process. Whitney (1972) uses the well known filtered Poisson process model for $C_k(t)$ to study various sampling strategies. One of the requirements of the National Primary Air Quality standards is to monitor a τ-hour average, $\tau = 1, 8, 24$, for some pollutants and different standards hold for each pollutant, e.g., one hour average of carbon monoxide concentration should not exceed 40,000 $\mu g/m^3$. Similarly, certain standards require the monitoring of the maximum of hourly averages. National primary and secondary standards are parts of the Clean Air Act. Essentially we are interested in the following quantities

$$\max_{0 < t < \tau} C_i(t) = a(\tau) \qquad (1)$$

$$\frac{1}{\tau}\int_0^{\tau} C_i(t)\, dt = b(\tau) \qquad (2)$$

Recent interest in the distribution of a (τ) and $b(\tau)$ has resulted in papers by Singpurwala (1972) and Barlow (1973). Assuming that concentrations at discrete points in time have a lognormal distribution, Singpurwala obtains results for the distribution of the maximum of the concentrations. It is interesting to note that the present investigator was one of the first few to propose lognormal models for pollutants and other trace substances in the human system, Rustagi (1964). This model has been confirmed by a large number of investigators recently for reference see Larsen (1971), Phinney and

Newman (1972), Eberhardt et. el. (1973), Hunt (1972), Vought and London (1964).

The available air pollution data is to be used for fitting the multivariate lognormal distribution for the pollutants. Although measurements are available at discrete points in time, $C_1(t)$ is a continuous process and the usual model for it should be a log Gaussian, rather than filtered Poisson process as suggested by Whitney. In proposing sampling strategies from monitoring point of view, one considers criterion (1) or (2). Restricted sampling for maxima can be done through probability sampling if the distribution of the maxima is known for sufficiently long periods of observation. Some such strategies for sampling a continuous process representing a contaminant so as to catch the maximum during certain fixed time with prescribed high probabilities are being considered. In the preliminary stages of the study, relevant problems in sampling theory were reviewed by Singh (1973).

REFERENCES

Barlow, Richard E. (1973). Averaging time and maxima for air pollution concentrations, Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability, Vol. VI. LeCam, L., Neyman, J., and Scott, E. (Editors). 433-442.

Eberhardt, L. L., Gilbert, R. O., Hollister, H. L., and Thomas, J. M., (1973). Sampling for contaminants in ecological systems, Report of Battelle-Northwest, Richland, Washington (Presented at the International Statistical Institute meetings in Vienna.)

Hunt Jr., William F. (1972). The precision associated with the sampling frequency of lognormally distributed air pollutant measurements, J. A. P. C. A.: 687-691.

Larsen, Ralph I. (1971). A mathematical model for relating air quality measurements to air quality standards, Environmental Protection Agency, Research Triangle Park, North Carolina.

Phinney, Dale E., and Newman, James E. (1972). The precision associated with the sampling frequencies of total particulate at Indianapolis, Indiana, J. A. P. C. A.: 692-695.

Rustagi, J. S., (1964). Stochastic behavior of trace substances, Archives of Environmental Health 8: 68-76.

Whitney, Cynthia (1972). A probabilistic model for assessing time varying contaminant levels, The Charles Stark Draper Laboratory, MIT, Cambridge, Mass.

Vought, R. L., and London, W. T., (1964). Iodine intake and excretion in healthy nonhospitalized subjects, American Journal of Clinical Nutrition 15: 124-132.

Singh, Daroga: On Repeat Survey sampling, Double sampling for Stratification and Non-sampling errors and bias in sample surveys, Seminar Reports, The Ohio State University, Division of Statistics 1973.

Research Activities

J. S. Rustagi participated in the Conferences on Statistical Aspects of Air Quality Data, held at the University of North Carolina, Chapel Hill, Nov. 19, 1972. He attended the Conference on Reliability and Biometry at Florida State University, Tallahassee, and was chairman of one of its sessions. During 1972, he was elected an Ordinary Member of the International Statistical Institute and attended its session held in Vienna, Austria during August 1973. He also visited Indian Statistical Institute during the summer of 1972.

Dr. Daroga Singh gave a series of seminars on (i) Repeat Survey Sampling, (ii) Double Sampling for Stratification (iii) Non-sampling Errors and Bias in Sample Survey, at The Ohio State University. He was invited to give Colloquium talks at University of Georgia, Athens, Ga., and Purdue University, Lafayette, Indiana during his stay at The Ohio State University.

PUBLICATIONS AND REPORTS

[1] Bai, Do Sun and Rustagi, J. S. Unbiased estimation and tests of independence in a Markov dependent firing distribution, To appear in Communications in Statistics.

[2] Gupta, R. S. and Singh, J. Paired Comparison Model Building and related inference,,Technical Report, Division of Statistics, The Ohio State University, 1973.

[3] Rustagi, J. S. Variational Methods in Statistics I - Classical Variational Methods, Technical Report, Division of Statistics, The Ohio State University, 1973.

[4] Rustagi, J. S. Variational Methods in Statistics II. Modern Variational Methods, Technical Report, Division of Statistics, The Ohio State University, 1973.

[5] Singh, J. Paired Camparison Ranking by linear programming, Communications in Statistics 1 (1973) 351-364

[6] Wong, Y. H. Sequential Estimation of Scale parameters of the Pareto Distribution, Communications in Statistics. 2 (1973); 145-154.

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